Student: ________________________________ Class: ______________________ Date:________________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

One factor that talent scouts look for in potentially competitive swimmers is the ratio of their height to their arm span. For most people, arm span is generally equal to height. Consider U.S. Olympic swimmer Michael Phelps, who is 6 feet, 4 inches (193 centimeters) tall with an arm span of 6 feet, 7 inches (200 centimeters). In fact, the U.S. swim team found that its male swimmers have an average height of 187.1 centimeters and an average arm span of 192.9 centimeters. Of course, other factors influence the success of a swimmer, but coaches often look at a swimmer's physical attributes, including arm span, to determine which strokes he or she should focus on.

At a local competitive swim club, the coach measured the height and arm span of his top 10 swimmers. The data are shown in the table below.

Height (cm)	Arm Span (cm)
172	173
173	175
179	182
180	185
183	187
186	189
187	186
190	195
191	191
192	196

Student: ___________________________________ Class: ________________________ Date:_________________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

1. Enter the data given in the table into your graphing calculator and make a scatterplot. Sketch the graph below and describe it in words.

2. Use what you know about the situation and the data to find a function model for this data set. Explain your reasoning.

3. Use your graphing calculator to compute a regression analysis of the swimmers' arm spans in relation to their height. What does the information from the calculator tell you? How does the equation given by the calculator compare to the function you found in Question 2?

4. Work in a group of four students. Each group member enters one of the data sets below into a graphing calculator, makes a scatterplot, and performs a linear regression analysis. Compare the graphs and the values of the correlation coefficients (***r***). Record an observation about how the value of ***r*** describes the strength and direction of the relationship between the variables.

x	*y*
-2	-4
-1	-2
0	0
1	2
2	4

x	*y*
-2	2
-1	-3
0	0
1	-2
2	5

x	*y*
-2	9
0	0
1	7
5	-2
7	4

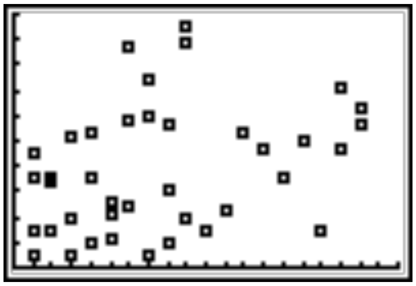
x	*y*
0	8
1	5
2	4
3	1
4	0

Student: ___________________________ Class: ___________________ Date: ______________

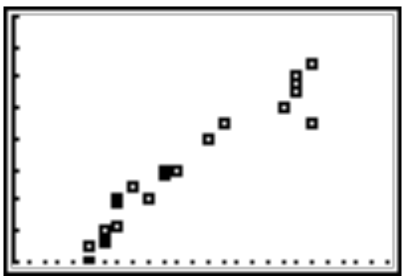
Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

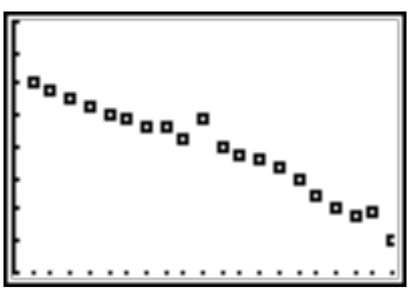
5. Consider each scatterplot below. Draw a line to match each ***r***-value to a scatterplot.



r = 0.972



r = 0.333



r = -0.976

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 1: Analyzing Linear Regression Equations

6. **REFLECTION**: Does a strong correlation indicate a cause-and-effect relationship between variables? Give examples to justify your response.

7. **EXTENSION:** Think of a situation that might have a linear relationship. Research the situation to find data relating the variables and perform a linear regression analysis on the data. Make sure your data set is of ample size. Use the regression analysis to determine a model and describe the strength of the model.

Student: ______________________ Class: ______________ Date: __________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 2: Comparing Linear and Exponential Functions

Coen sells magazine subscriptions. He is paid \$20 to start selling and then earns \$1.50 for each subscription he sells. The table shows the amount of money (***M***) Coen earns for selling ***n*** subscriptions.

n	***M***
0	\$20.00
1	\$21.50
2	\$23.00
3	\$24.50
4	\$26.00

1. In previous work, you wrote a linear function rule describing the amount of money Coen earns as a function of the number of subscriptions he sells. What do the domain and range of this situation represent?

2. Fill in the blanks below to find the differences between the given entries in the table. For each table, make a statement summarizing the relationship between changes in the domain and changes in the range.

n	***M***
0	\$20.00
1	\$21.50
2	\$23.00
3	\$24.50
4	\$26.00

n	***M***
0	\$20.00
1	\$21.50
2	\$23.00
3	\$24.50
4	\$26.00

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 2: Comparing Linear and Exponential Functions

3. Suppose Coen's earning structure changed so that for every magazine subscription he sold, he made 1.5 times his previous earnings. Again, assume that he starts with $20 for 0 subscriptions sold. Make a table showing Coen's earnings.

n	***M***
0	
1	
2	
3	
4	

4. In Question 2, you analyzed changes in the domain values and their impact on the values in the range. Now analyze the new data set you found in Question 3. Do these data show the same kind of "add-add" relationship as in the linear relationship in Question 2? Describe the effect on values in the range for this new set of data when values in the domain are changed incrementally by adding 1. Is this relationship the same when adding 2 to each domain value? Adding 5? Explain your answers.

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 2: Comparing Linear and Exponential Functions

5. **REFLECTION:** Describe a fundamental difference between linear and exponential functions based on a look at tables of values. How is the rate of change of a **linear function** different than the rate of change of an **exponential function**?

6. **EXTENSION:** Describe two additional "add-add" relationships that exist in real-world applications, and provide at least two representations of the relationships. Describe two additional "add-multiply" relationships that exist in real-world applications, and provide at least two representations of the relationships. Be prepared to share your examples with the class.

Student: ______________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

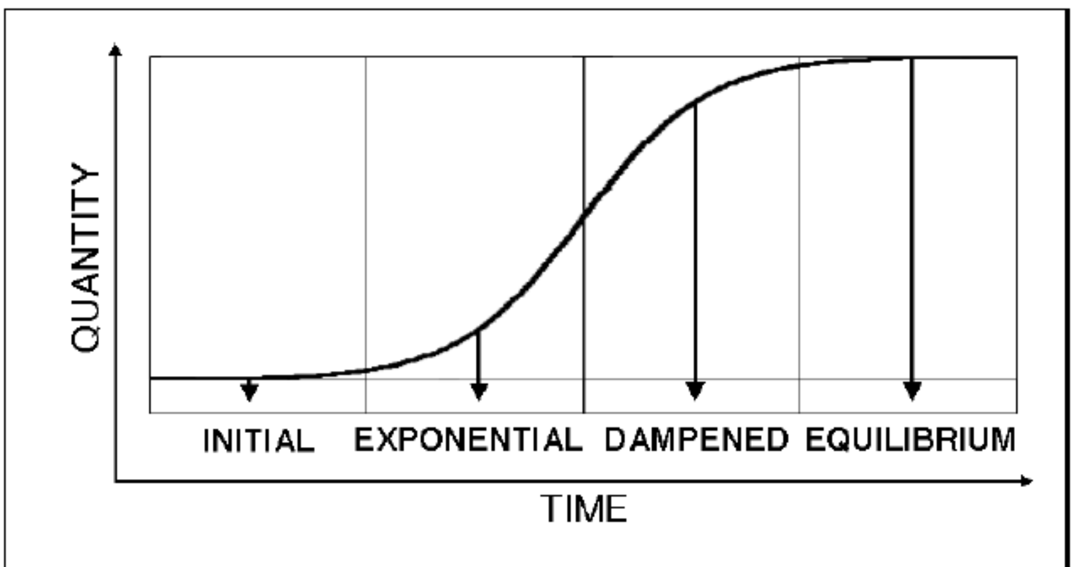
V.A Student Activity Sheet 3: Growth Model

H1N1—two letters and two numbers—are memorable as the most recent and perhaps greatest public health concern of this decade. The outbreak of this strain of influenza as most similar outbreaks can be simulated using mathematical techniques and models you are familiar with.

The simulation in this activity may create duplications or repetitions. For example, two people may both infect the same person. What are other possibilities of duplications or repetitions in a random number generating based simulation?

These duplications and repetitions are a desired aspect of the simulation because they signal the change from one stage of the simulation to the next stage.

The four stages of are labeled in the following graph. Remember the scenario you are considering here—the spread of the flu virus.



Intervals showing four stages of growth

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

1. What is happening with the spread of the flu virus in the graph on the previous page?

2. Use the following simulation procedure to complete the table on the next page. This simulates the introduction of the flu virus to a closed environment or population by means of a single infected individual.

Imagine a total population of 100 individuals. Each number from 0–99 in the Hundreds Chart represents an individual, with the number 0 used to portray the original host. Use the Hundreds Chart to keep track of the infected individuals by crossing off their number on the list as they become infected.

Hundreds Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Day 1: The original host infects a person represented by a randomly generated number. Generate a random integer between and including 0 and 99 using your graphing calculator or some other random number generating tool. Mark that person in the chart.

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Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

Day 2: The two infected people from Day 1 now infect two people, so generate two random integers.

Continue to simulate the rest of the days, completing the table of data up to Day 6.

Day	Number of initially infected people	Number of newly infected people	Total number of infected people
1	1	1	1
2	1		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

3. How is the number of infected people growing? What function would you use to model these data?

4. Make a scatterplot of the data from Days 1-6. Determine and record the model that best fits the data set. How do you know this model is best?

5. What are the independent and dependent variables in this model?

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

6. Graph your function rule over your scatterplot of Days 1-6 data. How well does the function rule fit your data?

7. Use your regression equation to predict the number of infected persons by Day 10. What conclusions can you draw from the data and predictions to this point?

8. Add Days 7-9 to the table of simulated data.

9. **REFLECTION:** What do you expect to occur as additional days are simulated? Why do expect this?

Using Functions in Models and Decision Making: Regression in Linear and Nonlinear Functions

V.A Student Activity Sheet 3: Growth Model

10. Complete the table, recording your simulations through Day 15.

11. Make a scatterplot of the day related to the total number of people infected with the flu virus.

12. You should recognize this graph from your work in the previous unit as the *logistic* graph. Use the regression capabilities of your graphing calculator to determine the function rule that best fits this data. Then graph this function rule over the scatterplot.

13. How well does the function rule fit the data?

14. **EXTENSION:** The graph of the *logistic function* displays *asymptotic behavior*. Investigate the meaning of an *asymptote* and describe why this graph in fact demonstrates this behavior. Describe another scenario where the data and resulting graph are similar to this type of graph and behavior.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

You may have noticed that during the winter the days are shorter and during the summer the days are longer. How much longer are days during the summer? Does the length of summer days change depending on the latitude of a place?

You will investigate these questions using data from four different cities at four different latitudes:

- Houston, Texas—30˚N latitude
- Philadelphia, Pennsylvania—40˚N latitude
- Winnipeg, Manitoba, Canada—50˚N latitude
- Porto Alegre, Brazil—30˚S latitude (addressed in Student Activity Sheet 5)

The data in the tables for this activity describe the length of daylight for the year 2009. The data table is based on two assumptions:

- The length of daylight is defined as the amount of elapsed time between sunrise and sunset.
- Because 2009 is not a leap year, there are 365 days in the year.

Which city would you expect to have more daylight during the summer, Houston or Philadelphia? Why do you think so?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part A: Houston

1. Make a scatterplot of the length of daylight by day number for Houston on the blank grid provided at the end of this activity sheet (Length of Daylight for Cities). To make the graph easier, make January 1 = Day 1 and December 31 = Day 365. In addition, graph the length of daylight in terms of minutes.

Date	Day Number	Houston	
		HH:MM	Min.
Jan. 1	1	10:17	617
Feb. 1	32	10:48	648
March 1	60	11:34	694
Apr. 1	91	12:29	749
May 1	121	13:20	800
June 1	152	13:57	837
July 1	182	14:01	841
Aug. 1	213	13:33	813
Sept. 1	244	12:45	765
Oct. 1	274	11:52	712
Nov. 1	305	11:00	660
Dec. 1	335	10:23	623

Source: U.S. Naval Observatory, www.usno.navy.mil

2. Enter the data into the stat lists of your graphing calculator. Use the calculator to make a scatterplot of the length of daylight by day number for Houston. Sketch your graph and describe your axes and scaling.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

3. Use your calculator to generate a sinusoidal regression model. Record the equation (round values to the nearest hundredth) in the Summary Table at the end of this activity sheet. Factor the value of b from the quantity ($bx - c$) and include that form of the equation as well.

4. Graph your model over your scatterplot. How well does the model fit your data?

5. Connect the points on your paper scatterplot with a smooth curve to represent the regression model.

6. Use your calculator to determine the maximum and minimum values for the length of daylight by day in Houston. Record these ordered pairs in your Summary Table and label them on your scatterplot. To which dates do these values correspond?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part B: Philadelphia

1. Make a scatterplot of the length of daylight by day number for Philadelphia. Plot the points on the same grid that you used for the Houston scatterplot.

Date	Day Number	Philadelphia	
		HH:MM	Min.
Jan. 1	1	9:23	563
Feb. 1	32	10:11	611
March 1	60	11:19	679
Apr. 1	91	12:41	761
May 1	121	13:56	836
June 1	152	14:46	886
July 1	182	14:57	897
Aug. 1	213	14:15	855
Sept. 1	244	13:03	783
Oct. 1	274	11:46	706
Nov. 1	305	10:28	628
Dec. 1	335	9:33	573

Source: U.S. Naval Observatory, www.usno.navy.mil

2. Enter the data for Philadelphia into a third list and graph the scatterplots for Houston and Philadelphia on the same screen. Sketch your graph and describe your axes and scaling.

3. Use your calculator to generate a sinusoidal regression model for the Philadelphia data. Record the equation (round values to the nearest hundredth) on the Summary Table. Factor the value of b from the quantity ($bx - c$) and include that form of the equation as well. Graph your model over your scatterplot. How well does the model fit your data?

4. Connect the points on your paper scatterplot with a smooth curve to represent the regression model for Philadelphia.

5. How do the regression models compare for Houston and Philadelphia?

Similarities:

Differences:

6. Use your calculator or graph to determine the maximum and minimum values for the length of daylight in Philadelphia. Record these ordered pairs in the Summary Table and label them on your paper scatterplot. To which dates do these values correspond?

7. How does the maximum length of daylight for Philadelphia compare to the maximum length of daylight for Houston?

8. **REFLECTION:** How does your answer to Question 7 compare to the prediction you made at the beginning of this activity?

9. Determine the intersection points of the regression models for Houston and Philadelphia. Record these ordered pairs in the Summary Table and label them on your scatterplot.

10. What do the intersection points mean in the context of this situation? ***Hint:*** Recall that your scatterplot shows the ordered pairs (Day Number, Length of Daylight) for Houston and Philadelphia.

11. **REFLECTION:** When is there more daylight in Houston than in Philadelphia? Is this what you expected? Why or why not?

When is there less daylight in Houston than in Philadelphia? Is this what you expected? Why or why not?

12. What is the difference in latitude between Houston and Philadelphia?

13. What is the difference in latitude between Philadelphia and Winnipeg?

14. What would you expect a scatterplot of length of daylight by day number for Winnipeg to look like? Why?

Part C: Winnipeg

1. Make a scatterplot of the length of daylight by day for Winnipeg. Plot the points on the same grid that you used for the other two scatterplots.

Date	Day Number	Winnipeg	
		HH:MM	Min.
Jan. 1	1	8:12	492
Feb. 1	32	9:23	563
March 1	60	11:01	661
Apr. 1	91	12:56	776
May 1	121	14:43	883
June 1	152	16:04	964
July 1	182	16:15	975
Aug. 1	213	15:11	911
Sept. 1	244	13:28	808
Oct. 1	274	11:37	697
Nov. 1	305	9:46	586
Dec. 1	335	8:25	505

Source: U.S. Naval Observatory, www.usno.navy.mil

2. Enter the data for Winnipeg into a fourth list and graph all three scatterplots on the same screen. Sketch your graph and describe your axes and scaling.

3. Use your calculator to generate a sinusoidal regression model for the Winnipeg data. Record the equation in your Summary Table (round values to the nearest hundredth). Factor the value of ***b*** from the quantity (***bx* - *c***) and include that form of the equation as well. Graph your model over your scatterplot. How well does the model fit your data?

4. Connect the points on your paper scatterplot with a smooth curve to represent the regression model for Winnipeg.

5. How do the regression models compare for all three cities?

   Similarities:

   Differences:

Using Functions in Models and Decision Making: Cyclical Functions
V.B Student Activity Sheet 4: Length of Daylight

6. Use your calculator to determine the maximum and minimum values for the length of daylight in Winnipeg. Record these ordered pairs in your Summary Table and label them on the paper scatterplot. To which dates do these values correspond?

7. Use your scatterplot to compare the points of intersection for all three graphs. What do they mean in the context of this situation?

8. The town of Seward, Alaska, is at 60°N latitude, just south of Anchorage, Alaska. What would you expect the length of daylight during the summer months to be in Seward compared to Winnipeg? The winter months?

9. What relationship do you think there is between a city's latitude and the amount of daylight it receives throughout the year?

10. REFLECTION: Describe how this application of sinusoidal regression and latitude as related to length of daylight is similar to the model of the Singapore Flyer. Compare and contrast the two situations with regard to similarities and differences of the model, scatterplot(s), and the functional relationship.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

Part D: Connections to Sinusoidal Functions

The parent function $y = \sin(x)$ can be transformed using four parameters. Each parameter describes a certain characteristic of the graph.

$y = A\sin[B(x - C)] + D$

- ***A*** represents the *amplitude* of the graph. The amplitude is the vertical distance from the horizontal axis of the graph to the maximum value or the minimum value of the graph. The amplitude is also equal to half of the difference between the maximum and minimum values.
- ***B*** represents the *angular frequency* of the graph. The angular frequency describes how many crests or troughs of the graph are present within a 360° or 2π portion of the domain of the graph. The angular frequency is also found by dividing 2π by the *period*, which is the horizontal distance between two consecutive maximum or minimum values.
- ***C*** represents the *phase shift*, or horizontal translation of a sine function.
- ***D*** represents a *vertical translation* of the graph. The line $y = D$ is the equation of the sinusoidal axis, which is the horizontal line representing the distance that is midway between the crests and troughs of the graph.

1. Look at the Houston row on the Summary Table. Subtract the maximum value of daylight from the minimum value of daylight, and then divide the difference by 2. How does this value compare to the amplitude (***A***) in the regression model?

2. Repeat the process of subtraction and division from Question 1 for Philadelphia and Winnipeg. What does this value suggest about the relationship between the maximum/minimum values and the amplitude for all three cities?

3. Divide 2π by the number of days in a year. How does the result compare to the angular frequency (***B***) for all three cities?

4. If the period of a sine function is the number of units before the cycle begins to repeat, why would the period of the regression model include a quotient with the number of days in a year?

5. How does the value for ***C*** (phase shift) in the factored form of your regression equations compare to the ***x***-coordinate of the first intersection points?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

6. How many minutes are there in 12 hours? Why would the vertical translation (***D***) be a number that is close to this value?

7. Why did the values of ***B***, ***C***, and ***D*** remain close to the same for the regression models for all three cities? Why did the value of ***A*** change for the models?

8. **EXTENSION:** Sun path diagrams show the path of the sun as it travels across the sky from sunrise to sunset at a given point on the surface of Earth. Because the sun's path varies each day, a sun path diagram reveals the part of the sky where the sun would be located for an observer on the ground at that point.

   Investigate cities at other latitudes, including those closer to the poles and the equator. Prepare a short presentation for the class.

   Some cities whose data can be obtained via the Internet (www.gaisma.com) include the following:

   - 80°N: Longyearbyen, Norway (78°N)
   - 70°N: Barrow, Alaska (71°N)
   - 60°N: Seward, Alaska; St. Petersburg, Russia; Anchorage, Alaska (61°N)
   - 20°N: Guadalajara, Mexico; Mexico City (19°N); Honolulu, Hawaii (21°N)
   - 10°N: Caracas, Venezuela; San Jose, Costa Rica
   - 0°: Quito, Ecuador; Kampala, Uganda; Pontianak, Indonesia
   - 10°S: Rio Branco, Brazil; Lima, Perú
   - 20°S: Belo Horizonte, Brazil; Port Hedland, Australia
   - 30°S: Durban, South Africa; Perth, Australia
   - 40°S: Valdivia, Chile; San Carlos de Bariloche, Argentina
   - 50°S: Stanley, Falkland Islands
   - 60°S: Villa Las Estrellas, Chilean Antarctic Territory

Student: ______________________ Class: ________________ Date:____________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight

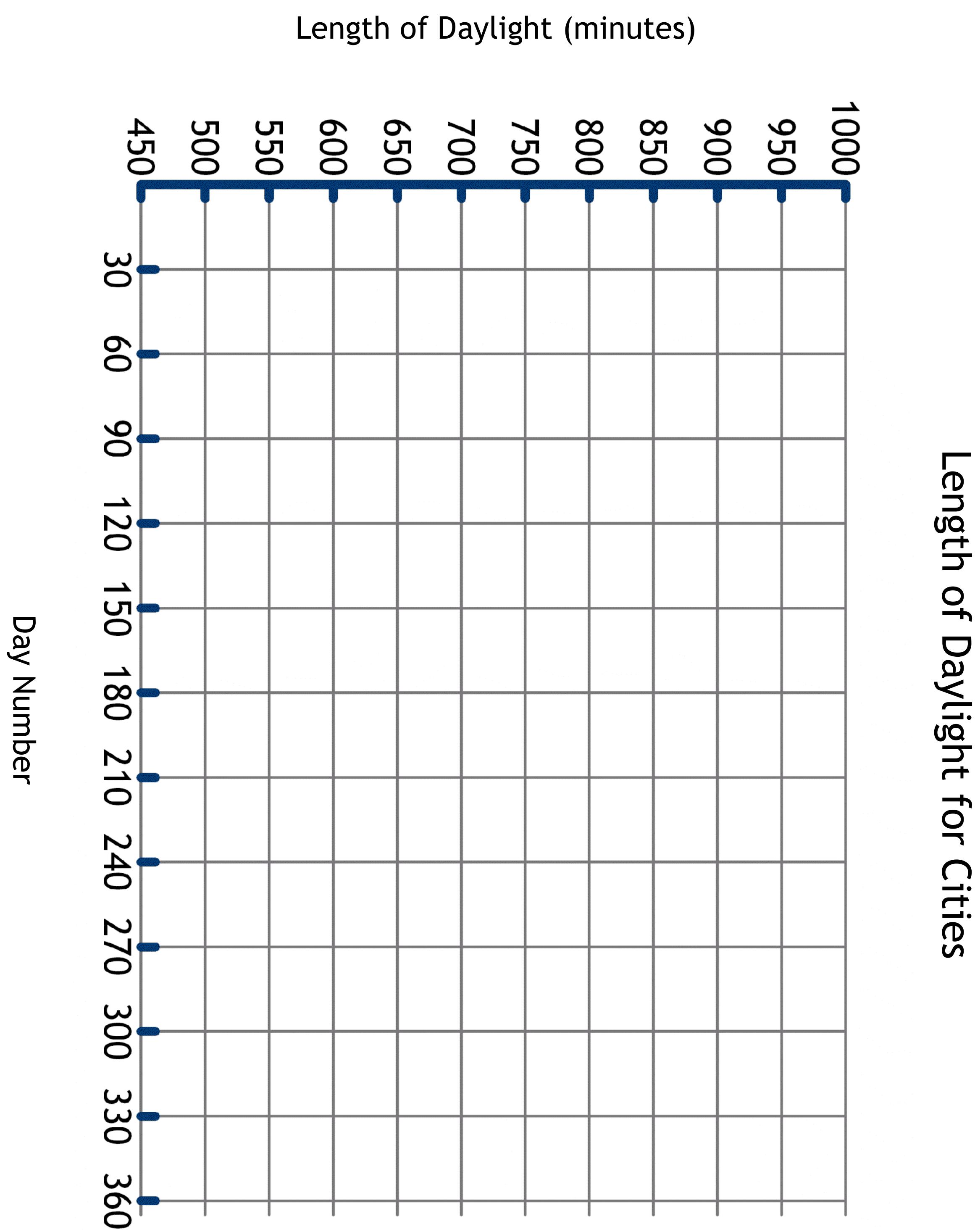
Summary Table for Length of Daylight

City	Regression Model	Maximum	Minimum	First Intersection	Second Intersection
Houston	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:		
Philadelphia	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:
Winnipeg	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:
Porto Alegre	Calculator form: Factored ***B***:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:	Ordered pair: Date: Length of day:

Student: ______________________________ Class: ____________________ Date: ______________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 4: Length of Daylight



Student: ___________________________ Class: ____________________ Date: ______________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

You investigated the relationship between a city's latitude and the length of daylight it experiences throughout the year. You did so by making scatterplots and finding regression models for the functional relationship between the day of the year and the length of daylight for three different cities at three different latitudes in the Northern Hemisphere:

- Houston, Texas—30°N latitude
- Philadelphia, Pennsylvania—40°N latitude
- Winnipeg, Manitoba, Canada—50°N latitude

In this activity, you will investigate the relationship between two cities that are the same distance from the equator, but on opposite sides of it: Houston, Texas, and Porto Alegre, Brazil.

Remember that the data in the tables for this activity describe the length of daylight for the year 2009 for each day. The data table is based on two assumptions:

- The length of daylight is defined as the amount of elapsed time between sunrise and sunset.
- Because 2009 is not a leap year, there are 365 days in the year.

You will need your Summary Table and scatterplots from Student Activity Sheet 4.

1. Porto Alegre, Brazil, is located in the Southern Hemisphere at 30°S latitude. Houston, Texas, is located in the Northern Hemisphere at 30°N latitude. How do you think the graphs of the length of daylight by day would compare for the two cities? Sketch your prediction, if needed, and explain why it might be true.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

2. Make a scatterplot of the length of daylight by day in Porto Alegre, Brazil. Plot the points on the same grid that you used for the scatterplots from the previous activity.

Date	Day Number	Houston		Porto Alegre	
		HH:MM	**Min.**	**HH:MM**	**Min.**
Jan. 1	1	10:17	617	14:03	843
Feb. 1	32	10:48	648	13:29	809
March 1	60	11:34	694	12:42	762
Apr. 1	91	12:29	749	11:45	705
May 1	121	13:20	800	10:55	655
June 1	152	13:57	837	10:19	619
July 1	182	14:01	841	10:15	615
Aug. 1	213	13:33	813	10:42	642
Sept. 1	244	12:45	765	11:30	690
Oct. 1	274	11:52	712	12:23	743
Nov. 1	305	11:00	660	13:17	797
Dec. 1	335	10:23	623	13:56	836

Source: U.S. Naval Observatory, www.usno.navy.mil

3. How does the scatterplot for Porto Alegre compare to the scatterplot for Houston? Does this match your prediction? Why do you think this is so?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

4. Use your calculator to generate a scatterplot of length of daylight by day for Houston. You may need to re-enter the data into your data lists. In addition, graph the regression equation that you found for Houston.

5. Enter the data for Porto Alegre into a third list and graph both scatterplots on the same screen. Sketch your graph and describe the axes and scaling.

6. Use your calculator to generate a sinusoidal regression model for the Porto Alegre data. Record the equation (round values to the nearest hundredth) in the Summary Table. Factor the value of b from the quantity ($bx - c$) and include that form of the equation as well.

7. Graph your model over your scatterplot. How well does the model fit your data?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

8. Connect the points on your paper scatterplot with a smooth curve to represent the regression model.

9. How do the regression models for Houston and Porto Alegre compare?

Similarities:

Differences:

10. Use your calculator to determine the maximum and minimum values for the length of daylight in Porto Alegre. Record these ordered pairs in the Summary Table and label them on your scatterplot. To which dates do these values correspond?

11. How does the maximum length of daylight for Porto Alegre compare to the maximum length of daylight for Houston?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 5: Crossing the Equator

12. How does the minimum length of daylight for Porto Alegre compare to the minimum length of daylight for Houston?

13. **REFLECTION:** Based on your observations of Porto Alegre and Houston, what would you conclude about the longest and shortest days for two cities on opposite sides of the equator?

14. Determine the intersection points of the regression models for Houston and Porto Alegre. Mark these points on your scatterplot and record them in your Summary Table.

15. What do the intersection points mean in the context of this situation? ***Hint:*** Recall that your scatterplot shows the ordered pairs (Day Number, Length of Daylight) for Houston and Porto Alegre.

16. How do the intersection points for the graphs of Houston, Philadelphia, Winnipeg, and Porto Alegre compare? What do these points mean in terms of the context of this situation?

17. Suppose you made a scatterplot of the length of daylight by day for Philadelphia (40°N latitude) and San Carlos de Bariloche, Argentina (40°S latitude). Based on what you noticed about the graphs for Houston and Porto Alegre, what would you expect the two scatterplots to look like?

18. **REFLECTION:** What generalization could you make about the relationship between the length of daylight over time for two cities that are the same distance from the equator but on opposite sides of it (like Houston and Porto Alegre)?

19. **EXTENSION:** What would you expect a scatterplot of the length of daylight by day to look like for a city like Quito, Ecuador, which lies on the equator? Why do you think this is so? Use the Internet to find data for Quito and test your conjecture.

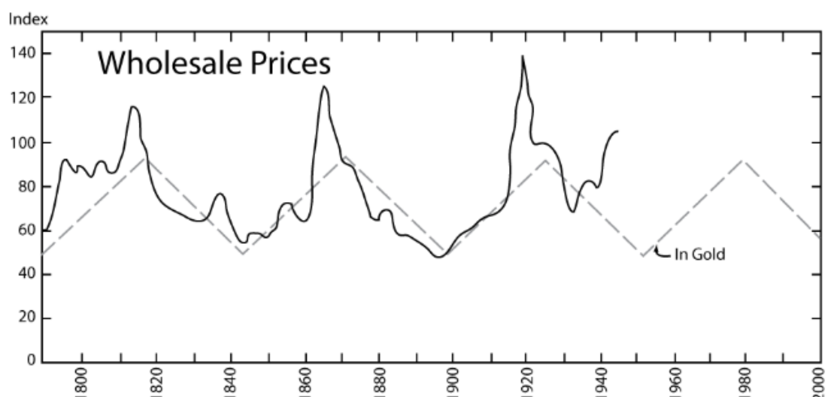
Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

Economists look for cycles to make predictions about the economy. Market traders also look for patterns in the prices of financial items such as stock, commodities, and currency value to make trades that yield the most money. In a cycle, prices rise and fall with a predictable regularity. If market traders can identify where in a cycle prices are, they can make decisions to increase their profit.

In 1947, economists Edward R. Dewey and Edwin R. Dakin published *Cycles—The Science of Predictions*, in which they identified a 54-year cycle in the wholesale price of goods. *Wholesale prices* are the prices that store owners pay the people who produce the goods (such as milk, gasoline, or chocolate chip cookies) to purchase the items to sell in their stores.

Dewey and Dakin presented a graph like the one shown below. The graph shows wholesale prices of goods in the United States in terms of a wholesale price index (WPI). The dashed line traces out the 54-year cycle that Dewey and Dakin describe.



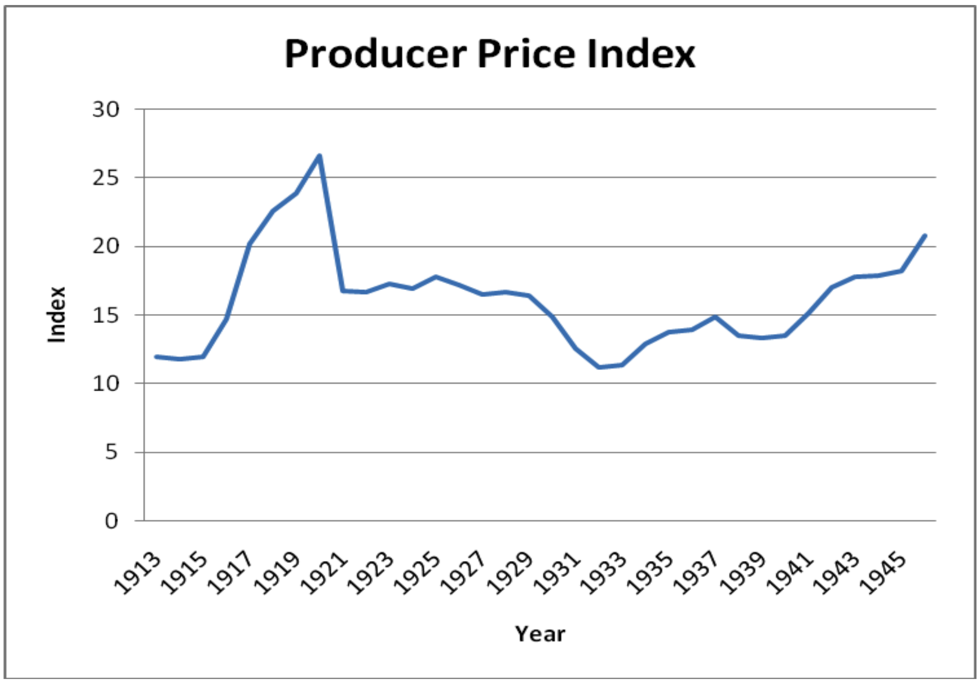
1. According to the graph, in what years do there appear to be *peaks*, or relative maximum values in the wholesale prices?

2. In what years do there appear to be *valleys*, or relative minimum values?

3. If there is a 54-year cycle between peaks and valleys, in what years should the next few maximum and minimum points occur?

In 1978, the U.S. Bureau of Labor Statistics (BLS) reclassified the WPI that Dewey and Dakin used into the Producer Price Index (PPI). In 1982, the BLS reset the benchmark for the PPI to 100.0 for the annual value of the PPI. As a result, historical data had to be recalibrated to be used for comparisons over time.

4. The graph shows the PPI as it was recalibrated using an index of 100.0 to represent the value for 1982. How does this graph compare to the one used by Dewey and Dakin for their 1947 book?



Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

5. The table at the right contains data from the BLS describing the commodity prices as measured by the PPI for certain years since 1940 (1982 = 100). Make a line graph of the PPI by year.

Year	PPI
1940	13.5
1944	17.9
1948	27.7
1952	29.6
1956	30.3
1960	31.7
1964	31.6
1968	34.2
1972	39.8
1976	61.1
1980	89.8
1984	103.7
1988	106.9
1992	117.2
1996	127.7
2000	132.7
2004	146.7
2008	189.7

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

6. Compare your scatterplot to the 54-year cycle described by Dewey and Dakin. Is there a maximum value where the Dewey and Dakin model predicts there to be one? Why or why not?

Is there a minimum value where the Dewey and Dakin model predicts there to be one? Why or why not?

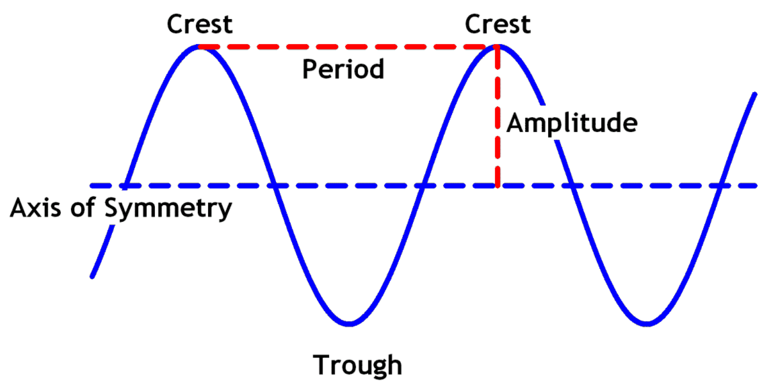
7. Does the trend in your scatterplot reveal the cyclical pattern Dewey and Dakin described in 1947?

Student: ________________________________ Class: ______________________ Date:________________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

Businesses use other cyclical models to describe seasonal phenomena. They refer to key attributes in cyclical models as shown below.



The *crest* is the maximum height of a wave, and the *trough* is the minimum height of a wave. The *period* is the distance between two consecutive crests or two consecutive troughs. The *axis of symmetry* is a horizontal line that runs exactly halfway between the crests and troughs. The *amplitude* is the distance between a crest or trough and the axis of symmetry.

8. Suppose that a particular business owner has determined that the function

$$\boldsymbol{y} = 200\sin(0.524(\boldsymbol{x} + 3.139)) + 400$$

can be used to determine the number of employees ($\boldsymbol{y}$) that he requires for month $\boldsymbol{x}$, where $\boldsymbol{x}$ = 1 corresponds to January 1.

Use your calculator to graph this function. Sketch your graph using the horizontal values from 1 to 12 and vertical values from 0 to 700.

9. EXTENSION: Recall that sine functions can be represented using the general form $y = A\sin(B(x - C)) + D$, where

- ***A*** represents the amplitude,
- ***B*** represents the angular frequency,
- ***C*** represents a factor of a horizontal translation, and
- ***D*** represents the vertical translation.

For this function, determine the values of ***A***, ***B***, ***C***, and ***D***.

A =

B =

C =

D =

10. Find the length of one cycle by dividing 2π by the frequency (***B***).

11. What is the vertical translation? Graph the line ***y*** = ***D*** on your graphing calculator.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

12. Determine the maximum and minimum values for number of employees. In what months do they occur?

13. How does the amplitude, combined with the vertical translation, describe the variation in number of employees needed for any given month?

14. When would you expect the next maximum value in the cycle to occur?

15. Change your viewing window so that you can see two full cycles of the graph, and determine the next maximum value. How does this compare with your prediction?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 6: Making Decisions from Cyclical Functions in Finance

16. Suppose the economic conditions change, and the business owner needs between 300 and 900 employees during the seasonal cycle. Which parameters should change? What should the new numbers be?

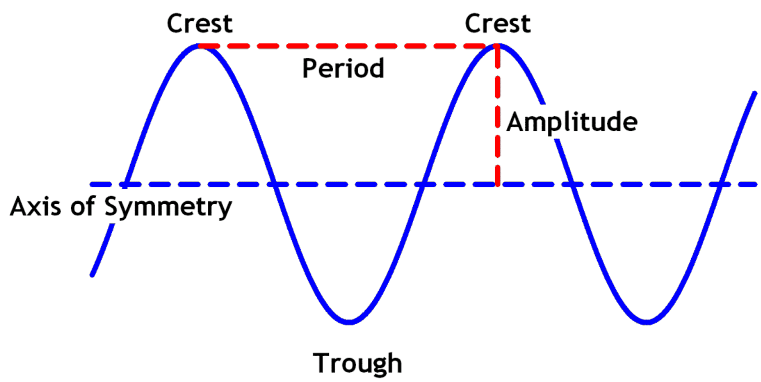
17. REFLECTION: What other types of employment might be cyclical in nature?

Student: ________________________________ Class: ______________________ Date:________________

Using Functions in Models and Decision Making: Cyclical Functions

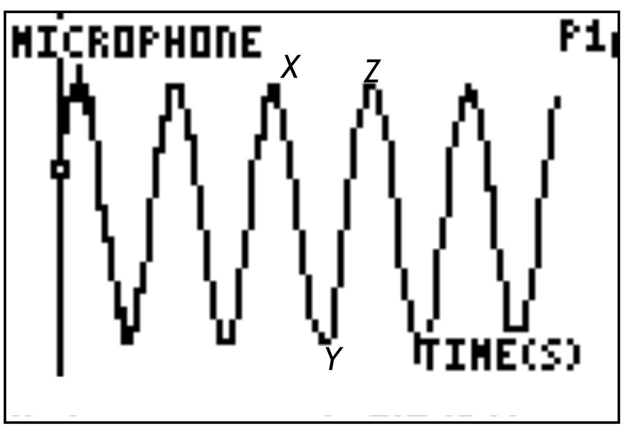
V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

Recall from your science class that sound travels in waves. A wave has several important parts:



The *crest* is the maximum height of a wave, and the *trough* is the minimum height of a wave. The *period* is the distance between two consecutive crests or two consecutive troughs. The *axis of symmetry* is a horizontal line that runs exactly halfway between the crests and troughs. The *amplitude* is the distance between a crest or trough and the axis of symmetry.

Mr. Licefi's math class used a calculator-based laboratory (CBL) and a microphone to collect the following sound data. Notice that Points *X*, *Y*, and *Z* are labeled in the graph.



X (0.0054, 6.5)

Y (0.0065, 2.5)

Z (0.0076, 6.5)

1. If *X* and *Z* each represent a crest, what is the period of the sound wave? (Do not forget your units!)

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

2. The frequency of a sound wave can be found by taking the reciprocal of the period. What is the frequency of this sound wave? The unit for frequency is hertz.

3. If *B* represents a trough, what is the amplitude of the sound wave?

4. In a sound wave, the frequency represents the pitch of the sound, and the amplitude represents the volume. For the sound wave that Mr. Licefi's class measured, what is the pitch and volume?

5. What amplitude is required to produce a sound wave that is twice as loud?

6. What are the domain and range of the function that models the sound wave?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

7. If the sound that Mr. Licefi's class measured lasted for 8 seconds and stayed the same pitch (from Question 4), what are the domain and range of the sound wave?

8. Compare the domain and range for the function that models the sound wave and the domain and range for the sound wave itself. Explain any similarities or differences.

Student: ________________________________ Class: ______________________ Date:_______________

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

Mrs. Kline's economics class was studying a data set that gives the price per pound of ground beef for the month of January from 1980 to 1996.

Year	Year Number	Cost (dollars)
1980	0	1.821
1981	1	1.856
1982	2	1.794
1983	3	1.756
1984	4	1.721
1985	5	1.711
1986	6	1.662
1987	7	1.694
1988	8	1.736
1989	9	1.806
1990	10	1.907
1991	11	1.996
1992	12	1.926
1993	13	1.970
1994	14	1.892
1995	15	1.847
1996	16	1.799

Source: U.S. Bureau of Labor Statistics

9. Use your graphing calculator to make a scatterplot of cost by year number.

10. Does the data set appear to be cyclical? Explain your reasoning.

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

11. An economics textbook suggests that the function $\boldsymbol{y} = 0.169\sin[0.52(\boldsymbol{x} + 2.78)] + 1.82$ can be used to model the data approximately. Graph this function over your scatterplot to verify that suggestion. Describe the axes and scaling, and sketch your graph.

12. **EXTENSION:** Recall that sine functions can be represented using the general form $\boldsymbol{y} = \boldsymbol{A}\sin(\boldsymbol{B}(\boldsymbol{x} - \boldsymbol{C})) + \boldsymbol{D}$, where

- ***A*** represents the amplitude,
- ***B*** represents the angular frequency,
- ***C*** represents a factor of a horizontal translation, and
- ***D*** represents the vertical translation.

For this function, determine the values of ***A*, *B*, *C*,** and ***D***.

A =

B =

C =

D =

13. Find the length of one cycle by dividing 2π by the frequency (***B***).

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

14. How well does the suggested function model the data?

15. Use the regression equation to predict the cost per pound of ground beef in January 2009.

16. Use the Internet to determine the actual cost per pound of ground beef in January 2009.

17. How well did your model predict the cost of ground beef in January 2009? Why do you think the model performed this way?

Using Functions in Models and Decision Making: Cyclical Functions

V.B Student Activity Sheet 7: Making Decisions from Cyclical Functions in Science and Economics

18. REFLECTION: What can you say about using a cyclical model to predict values beyond a given data set?

OR

How well could ocean waves be modeled using a sinusoidal function?

19. EXTENSION: What other natural or business phenomena could be modeled using a cyclical model? How well do you think those models could predict future values?

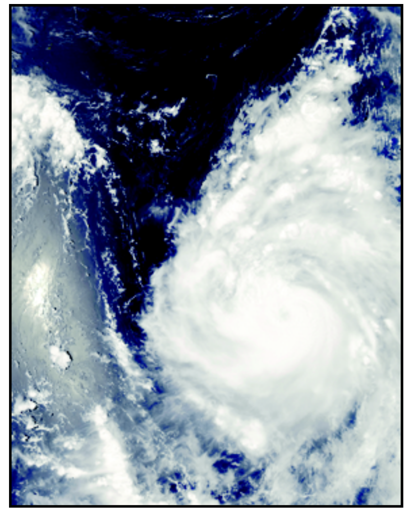
OR

Using a CBL and a microphone probe, capture your own data from sound waves that you generate. Then compare these data to the data used in the lesson.

Student: ______________________ Class: ________________ Date: ____________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions



Texas experiences a wide variety of weather, including hurricanes. Coastal residents often feel the direct effects of hurricanes when they make landfall along the coast. Cities and towns that are directly hit by a hurricane can sometimes take years to rebuild. Galveston is one such city.

Galveston was almost completely destroyed by the storm that hit in 1900, the deadliest hurricane in U.S. history. Rebuilding after the storm took several years, partly because residents raised the elevation of the entire city and built the Galveston Seawall to protect the city. Other towns were not so resilient. In 1886, residents of Indianola completely abandoned the ruins of their town on the shores of Matagorda Bay after it was wiped away by a strong hurricane.

Meteorologists use the Saffir-Simpson scale to describe the strength of a hurricane. This scale is based on a combination of wind speed and barometric pressure. The faster the wind speed and the lower the barometric pressure, the higher the rating of the hurricane on the Saffir-Simpson scale.

Saffir-Simpson Scale

Category	Wind Speed (miles per hour)
1	74–95
2	96-110
3	111-130
4	131-155
5	156 and above

Many hurricanes have struck the Texas coast, but there have been no recorded Category 5 hurricanes, which are the strongest, most destructive storms. Although many Caribbean and Central American nations have been pounded by Category 5 hurricanes, the United States has been hit by only three: the 1935 Labor Day Hurricane, which struck the Florida keys; Hurricane Camille, which struck Pass Christian, Mississippi, in 1969; and Hurricane Andrew, which struck near Homestead, Florida, in 1992.

Student: ____________________ Class: ____________________ Date: ____________________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

The following table shows the year, wind speed, and Saffir-Simpson category for some hurricanes that have made landfall on the Texas coast. This table also includes the Category 5 storms that have hit the United States.

Texas Hurricanes

Hurricane	Year	Wind Speed (miles per hour)	Category
Indianola Storm	1886	155	4
Galveston Storm	1900	125	3
Brownsville Storm	1933	100	2
Labor Day Storm*	1935	161	5
Audrey	1957	100	2
Debra	1959	105	2
Carla	1961	150	4
Beulah	1967	140	4
Camille*	1969	190	5
Celia	1970	130	3
Allen	1980	115	3
Alicia	1983	115	3
Bonnie	1986	86	1
Andrew*	1992	167	5
Bret	1999	115	3
Claudette	2003	90	1
Rita	2005	115	3
Dolly	2008	86	1
Ike	2008	110	2

*Storm did not make landfall in Texas.
Source: National Hurricane Center

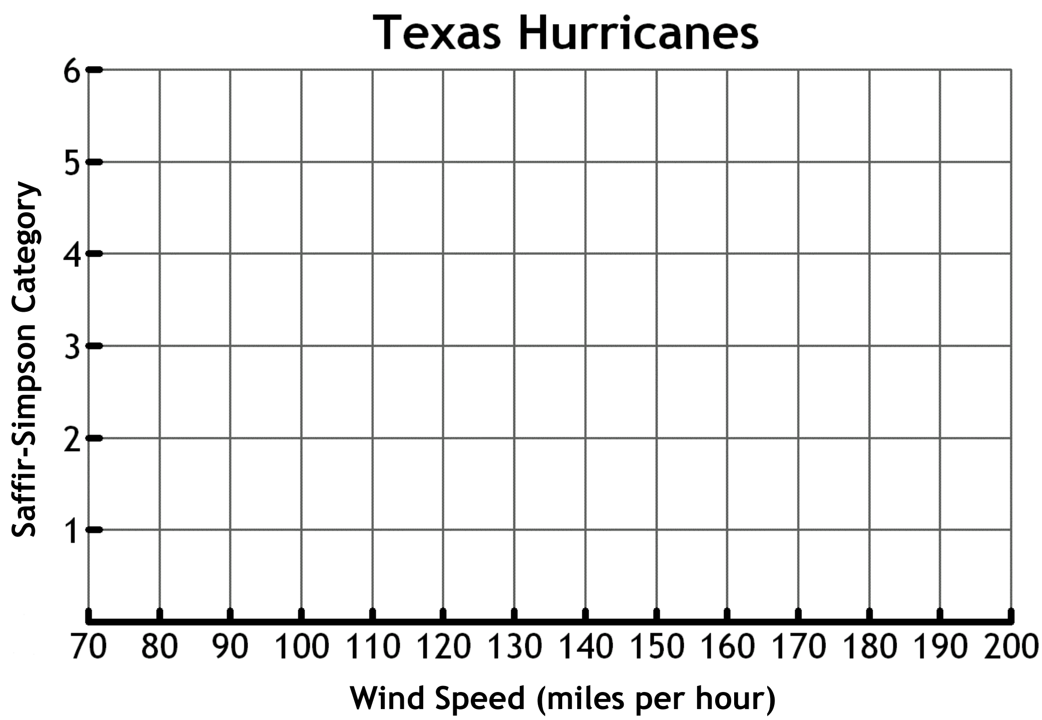
Student: ______________________ Class: ________________ Date: ____________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

1. Write a dependency statement that describes the relationship between the two variables, wind speed and Saffir-Simpson category.

2. Make a scatterplot of the Saffir-Simpson category versus wind speed for the hurricanes listed in the table.



3. Now mark the wind speed endpoints for each Saffir-Simpson category on the scatterplot. Connect those endpoints with a line segment. For example, along the line for Category 1, mark the wind speeds 74 and 95 [that is, the points (74, 1) and (95, 1)] and then connect them with a line segment.

4. Is it possible for a hurricane to be rated between Category 1 and Category 2? Why or why not?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

Hurricane wind speeds are difficult to measure precisely. Thus, most hurricane wind speeds are estimated to the nearest 5 miles per hour. Suppose a new technology were invented that allowed meteorologists to measure hurricane wind speeds very precisely.

5. If a hurricane had a wind speed of 95.1 miles per hour, what category would it be rated? How do you know?

6. Revise the Saffir-Simpson scale so that you can rate hurricanes with wind speeds that lie between the existing categories.

Revised Saffir-Simpson Scale

Category	Wind Speed (miles per hour)
1	
2	
3	
4	
5	

7. When graphing inequalities, how do you represent an endpoint that does not include *or equal to*?

8. Use a closed or open endpoint to revise your scatterplot for the new hurricane rating scale.

9. What kind of function does your new scatterplot represent?

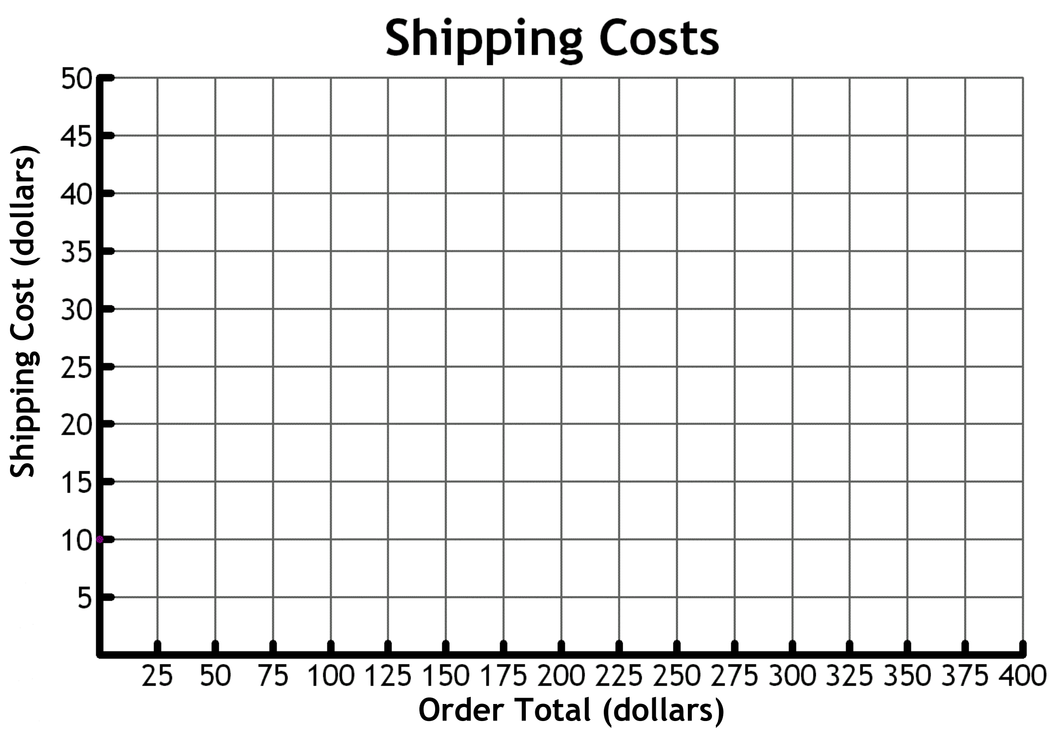
Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

An online store uses a step function to determine shipping costs.

Order Total	Shipping Costs	
	Continental United States	**Europe**
Less than \$25.00	\$5.00	\$10.00
\$25.00-\$74.99	\$10.00	\$20.00
\$75.00-\$124.99	\$15.00	\$30.00
\$125.00-\$349.99	\$20.00	\$40.00
\$350.00 and greater	\$25.00	\$50.00

10. Use a colored pencil to make a graph of shipping costs versus the order total.



Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

11. For orders shipped to Europe, the shipping cost for the United States is doubled. Fill in the table to show the shipping costs to Europe. Then use a different colored pencil to make a graph of the shipping costs to Europe versus the order total.

12. How do the two graphs compare?

13. **REFLECTION:** How do step functions compare to linear functions?

14. **REFLECTION:** How is multiplying a step function by a constant multiplier similar to multiplying the slope of a linear function by a constant multiplier?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 8: Introducing Step and Piecewise Functions

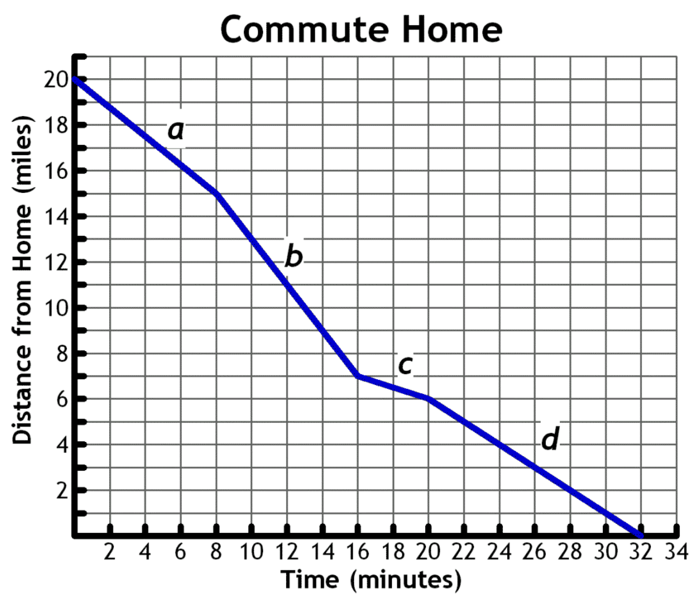
15. **EXTENSION:** What other situations can be modeled using a step function? Use the Internet to collect data and generate a graph of a situation. How does your graph compare to those in this activity?

Student: ___________________________ Class: ___________________ Date:_____________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

Mrs. Washington lives 20 miles from her office and drives her car to and from work every day. The graph below shows her distance from home over time as she drove home from work one day.



1. Write a dependency statement expressing the relationship between the two variables, distance and time.

The following table will be used to answer Questions 2, 6, and 8.

Segment	Slope	Equation of Line	Domain	Range
a				
b				
c				
d				

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

2. Find the slope of each line segment in the graph of Mrs. Washington's commute. Record your results in the table.

3. How did you find the slope of each segment?

4. What does the slope of a line segment represent in the context of this situation?

5. Is the slope an increasing or decreasing rate of change? What does this mean in the context of this situation?

6. Find the equations of the four line segments in the graph. Record your results in the table.

7. How did you determine the equations of the lines?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

8. Identify the domain and range of the line that describes each segment of Mrs. Washington's commute. Use inequality symbols to indicate the domain and range, and record your results in the table.

9. Graph the line that represents Segment ***a*** in your graphing calculator. To do this, set your viewing window to match the graph at the beginning of the activity.

10. Now, restrict the domain of the line. If possible, use graphing technology. Sketch your graph. Explain why the graph looks like it does.

11. Graph the line that represents Segment ***b***. Restrict the domain of the line as needed. What do you expect the graph to look like? Sketch your prediction before you actually draw or display the graph.

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

12. How does your prediction compare with what the graph looks like? Explain any differences.

13. Repeat the procedure to graph the lines for Segment ***c*** and Segment ***d***. Sketch your final graph.

14. What piece of information did you need to enter into the calculator to tell it which parts of the four lines it should graph?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

15. REFLECTION: Describe earlier types of functions that can be analyzed using the terminology used with step and piecewise functions. Give an example of an application of the function.

Student: ______________________ Class: ________________ Date: ____________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

The height of a diver above a body of water as a function of time can be given using two different functions: a constant function for the time the diver is on the diving board and a quadratic function for the time when the diver jumps off the board and falls toward the water.

Rafael is on vacation with his family in Acapulco, Mexico. La Quebrada is a famous cliff that is about 35 meters above the ocean surface. For many years, divers have jumped off La Quebrada into the Pacific Ocean. Rafael has signed up to go cliff diving.

16. Rafael stands on the cliff, 35 meters above the ocean surface below. What function describes his height above the ocean surface (***h***) as a function of time (***t***) while he stands on the cliff?

Rafael is next. He walks to the edge of the cliff and stands still for 3 seconds. Then he dives off the cliff. As soon as he leaves the cliff, his height above the ocean surface can be found using the function $\boldsymbol{h} = -4.9(\boldsymbol{t} - 3)^2 + 35$, where ***h*** represents Rafael's height from the ocean surface and ***t*** represents the time since Rafael stood at the edge of the cliff.

17. Fill in the table below to describe Rafael's height above the ocean surface over time.

	Function, *h(t)*	**Domain**
Standing still		
Free-fall motion		

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 9: Another Piecewise Function

18. Use the domain restrictions to graph Rafael's height above the ocean surface over time on your graphing calculator, if possible. Describe the domain, range, and scaling and sketch the graph.

19. **EXTENSION**: What other situations could be modeled using piecewise functions like the ones used to describe Mrs. Washington's commute or Rafael's cliff-diving experience? Investigate one of the situations and prepare a brief report for the class regarding your findings.

Student: ______________________________ Class: ____________________ Date: ______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

Part A

Have you ever taken a medication that your doctor warned you would not take effect for a few days? In this activity, you will investigate why that is the case.

Consider the allergy medicine Sneeze-B-Gone. The regular adult dose is 20 milligrams. As with all medicines, the body gradually filters Sneeze-B-Gone out of the bloodstream. The rate at which the medicine is filtered out is called the *flush rate*. For Sneeze-B-Gone, the flush rate is 30%. In other words, 24 hours after the pill is taken, 30% of Sneeze-B-Gone has flushed out of the body.

1. If 30% of Sneeze-B-Gone has flushed out of the body after 24 hours, what percent of Sneeze-B-Gone remains?

2. Use your calculator's recursion feature to fill in the table below, assuming that an adult is taking one 20-milligram dose per day.

3. At what value does the amount of Sneeze-B-Gone in the bloodstream level off? How many days does it take for that to happen?

4. What type of function could model the amount of Sneeze-B-Gone in the bloodstream as a function of time? Explain your choice.

Day	Sneeze-B-Gone in Bloodstream (in mg)
1	20
2	34
3	43.8
4	
5	
6	
7	
8	
9	
10	

Day	Sneeze-B-Gone in Bloodstream (in mg)
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

5. What would you expect a graph of the amount of Sneeze-B-Gone in the bloodstream as a function of time to look like? Explain your prediction.

6. Recall that the general form for exponential decay functions is $\boldsymbol{y} = \boldsymbol{a}(\boldsymbol{b})^{\boldsymbol{x}}$, where $\boldsymbol{a}$ represents the starting amount of the substance and $\boldsymbol{b}$ represents the rate of decay. For a 20-milligram dose and a 30% flush rate, what exponential function could describe the amount of Sneeze-B-Gone in the bloodstream ($\boldsymbol{y}$) as a function of time ($\boldsymbol{x}$)? (Do not forget that $\boldsymbol{b}$ represents the percent of Sneeze-B-Gone that remains in the bloodstream.)

7. Since the patient did not begin taking the medicine until Day 1, adjust your function rule by subtracting 1 from the exponent. Graph the function on your graphing calculator. Sketch your graph and describe your viewing window.

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

8. If time (***x***) is given in terms of the number of days, what happens to the amount of Sneeze-B-Gone in the patient's bloodstream at the start of Day 2 when the patient takes a second pill? How does this affect the graph?

9. Use what you learned about step and piecewise functions in previous activities to restrict the domain of the graph. Sketch your new graph.

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

10. For Day 2, enter the function $\boldsymbol{y} = 34 \bullet 0.7^{x-1}$ into your calculator. What do the constants 34, 0.7, and 2 represent? Sketch the new graph.

11. Based on the functions for Day 1 and Day 2, write a function from the data in your table for Day 3 and a function for Day 4.

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

12. Graph both of these new functions. What patterns do you notice? What do you expect the graph for Day 5 to look like?

13. Test your prediction by writing a function for Day 5.

14. **REFLECTION:** Assume the patient takes 20 milligrams of Sneeze-B-Gone every day. If you extend the graph to Day 20 or beyond, what would the minimum amount of Sneeze-B-Gone in the bloodstream be? The maximum amount?

Student: ______________________ Class: ________________ Date: ____________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 10: Concentrations of Medicine

Part B

1. Suppose a patient requires a 30-milligram dose of Sneeze-B-Gone. Use home screen recursion on your calculator to fill in the table.

2. At what value does the amount of Sneeze-B-Gone in the bloodstream level off? How many days does it take for that to happen?

3. How does the function rule for the 20-milligram dose change for a 30-milligram dose? Write the new function rule for the portion of the graph between Day 1 and Day 2.

Day	**Sneeze-B-Gone in Bloodstream (in mg)**
1	30
2	
3	
4	
5	
6	
7	
8	
9	
10	

Day	**Sneeze-B-Gone in Bloodstream (in mg)**
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

4. How do you think those changes would affect the graph of the new function rule?

Using Functions in Models and Decision Making: Step and Piecewise Functions

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5. Use your graphing calculator to test your prediction. Sketch your graph.

6. When the amount of Sneeze-B-Gone in the bloodstream levels off for a patient taking a 30-milligram daily dose, what are the minimum and maximum amounts of Sneeze-B-Gone in the bloodstream within a given day?

7. Suppose a patient requires a 40-milligram dose of Sneeze-B-Gone. Based on what you have observed so far, what would you expect the function rule and graph to look like?

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8. Use recursion on your calculator to fill in the table.

9. At what value does the amount of Sneeze-B-Gone in the bloodstream level off? How many days does it take for that to happen? You may need to extend the values in the table.

10. How does the function rule for the 30-milligram dose change with a 40-milligram dose? Write the new function rule for the portion of the graph between Day 1 and Day 2.

Day	Sneeze-B-Gone in Bloodstream (in mg)
1	40
2	
3	
4	
5	
6	
7	
8	
9	
10	

Day	Sneeze-B-Gone in Bloodstream (in mg)
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

11. How do you think those changes would affect the graph of the new function rule?

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12. Use your graphing calculator to test your prediction. Sketch your graph.

13. When the amount of Sneeze-B-Gone in the bloodstream levels off for a patient taking a 40-milligram dose, what are the minimum and maximum amounts of Sneeze-B-Gone in the bloodstream within a given day?

14. **REFLECTION:** How does an increase in dose affect the amount of Sneeze-B-Gone in the bloodstream when the amount levels off?

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15. Fill in the table below. What relationships do you notice?

Dose	**Flush Rate**	**Leveled-off Amount**	$\frac{\textbf{Dose}}{\textbf{Flush Rate}}$
20			
30			
40			

16. **REFLECTION:** If you were a doctor or nurse and you knew that a patient needed to have about 100 milligrams of Sneeze-B-Gone in his bloodstream for the medicine to be effective, what dose would you prescribe? Explain your decision.

17. **EXTENSION:** A new cholesterol-lowering medicine has a flush rate of 50%. For a 20-milligram dose of this medicine, how do the function rules and graph compare to those for the 20-milligram dose of Sneeze-B-Gone with a flush rate of 30%? Use your graphing calculator to investigate. Present your work to the class.

Student: ______________________________ Class: ____________________ Date:______________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

When electricity became widely distributed during the early part of the 20th century, state governments regulated the electricity industry as a monopoly. One electric company had the rights to generate and distribute electricity for a city or a certain part of the state. In return, the government laid out a set of rules for what the electric company could and could not do.

During the 1990s and early 21st century, many states deregulated electricity. As a result, numerous electric companies can now provide electricity for a particular area. One such company is Lights and Power. To attract customers, Lights and Power is advertising a special:

Cheapest Electricity in Town!

To 1,000 kWh—$0.11 per kWh
More than 1,000 to 1,500 kWh—$0.18 per kWh
More than 1,500 kWh—$0.25 per kWh

No hidden fees! We promise!

1. According to the advertisement, how much does the first 1,000 kilowatt-hours (kWh) of electricity cost a customer?

2. Suppose Mrs. Brown uses 1,200 kilowatt-hours of electricity. How much does she pay for the first 1,000 kilowatt-hours?

   How much does she pay for the next 200 kilowatt-hours of electricity?

   How much does she pay altogether for 1,200 kilowatt-hours of electricity?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

3. Use the information in Lights and Power's advertisement to determine the cost of electricity for the amounts listed in the table.

Amount of Electricity (kWh)	Process	Cost ($)
700	700(0.11)	77
800		
900		
1,000		
1,100		
1,200	1,000(0.11) + (1,200 – 1,000)(0.18)	146
1,300		
1,400		
1,500		
1,600		
1,700		
1,800		
1,900		

4. Write an equation to describe the cost (***y***) of the number of kilowatt-hours of electricity (***x***) to 1,000 kilowatt-hours.

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5. For what domain does your function model the cost of the first 1,000 kilowatt-hours of electricity?

6. Write an equation to describe the cost (c) of the number of kilowatt-hours of electricity (x) from 1,001 to 1,500 kilowatt-hours.

7. For what domain does your function model the cost of 1,001 to 1,500 kilowatt-hours of electricity?

8. Write an equation to describe the cost (m) of the number of kilowatt-hours of electricity (x) more than 1,500 kilowatt-hours.

9. For what domain does your function model the cost of more than 1,500 kilowatt-hours of electricity?

Using Functions in Models and Decision Making: Step and Piecewise Functions

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10. Write three piecewise functions, including limitations on the domain, that describe the cost of purchasing electricity from Lights and Power.

11. Use your graphing calculator to make a scatterplot of cost versus amount of electricity. Describe the axes and scaling and sketch your graph.

12. Graph your piecewise functions over your scatterplot. Use the domain restrictions. How well do the functions model the data generated by the electricity plan?

Using Functions in Models and Decision Making: Step and Piecewise Functions

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13. The function $y = 0.11x$ has a domain of all real numbers. Why is the domain of the function as it is applied in this situation restricted?

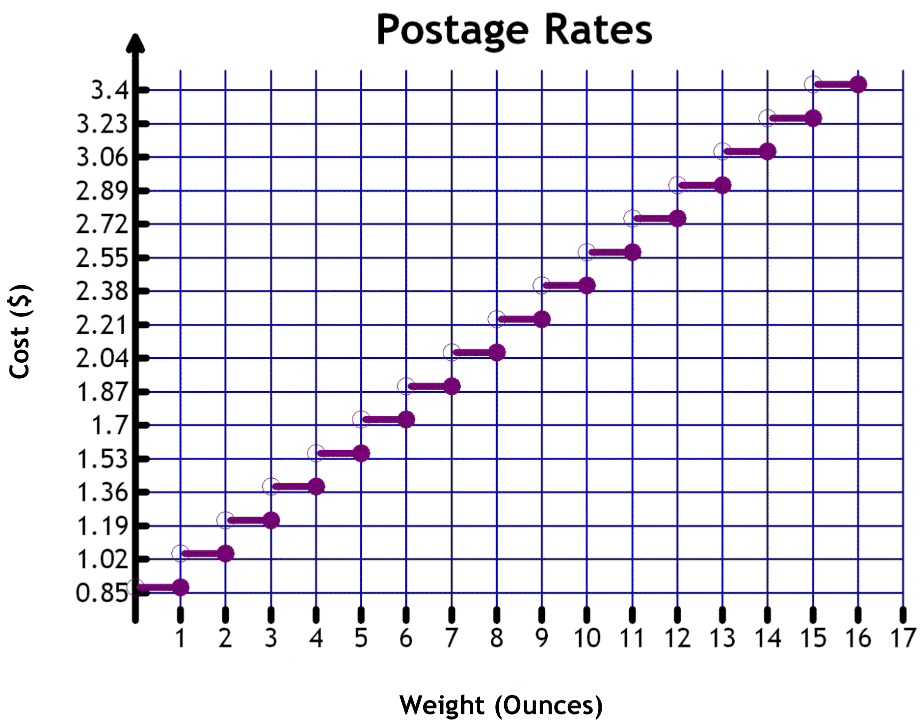
Student: ________________________________ Class: ______________________ Date:________________

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

As of May 2009, the U.S. Postal Service adjusted its rates so that mailing a large envelope costs $0.88 for the first ounce and $0.17 for each additional ounce. There is a weight limit for all first-class mail—letters and parcels mailed first class cannot exceed 13 ounces.

Consider the graph below.



14. What type of function is represented by the graph? How do you know?

15. Is this type of function appropriate to represent the U.S. Postal Service rates for sending large envelopes by first-class mail? Why or why not?

Using Functions in Models and Decision Making: Step and Piecewise Functions

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16. How well does the graph represent the U.S. Postal Service rates for sending large envelopes by first-class mail? How do you know?

17. How could you modify the graph to better represent the situation?

18. **REFLECTION:** What types of situations can a step function be used to model?

Using Functions in Models and Decision Making: Step and Piecewise Functions

V.C Student Activity Sheet 11: Making Decisions from Step and Piecewise Models

19. REFLECTION: How are step functions similar to piecewise functions? How are they different?

20. EXTENSION: Research taxicab fares for your city or a city that you want to visit. What type of function is most appropriate to represent those fares? Generate a graph to show the fares and present your findings to the class.

21. EXTENSION: Research to determine an appropriate response to the following question. Prepare a short presentation of your findings.

Would federal income taxes be better modeled with a step function or a piecewise function?